Reading Time: An initial **2 minutes** to view **BOTH** sections

MATHEMATICS METHODS : UNITS 3 & 4, 2021

Test 1 – (10%)

3.1.1 to 3.1.16

Time Allowed 20 minutes	First Name	Surname	Marks 20 marks

Circle your Teacher's Name: Mrs Alvaro, Mrs Bestall, Ms Chua, Mr Gibbon, Mrs Greenaway, Mr Luzuk, Mrs Murray, Ms Robinson, Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Not Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART A – CALCULATOR FREE

QUESTION 1 **[7 marks]**

Differentiate the following, simplifying fully.

a) $f(r) = \frac{r+1}{r-1}$ **[2 marks]**

$f'(r) = \frac{(r-1).1 - [(r+1).1]}{(r-1)^2}$ ✓ applies and substitutes correctly into Q.R.

$= \frac{r-1-r-1}{(r-1)^2}$

$= -\frac{2}{(r-1)^2}$ ✓ correct and simplified.

b) $f(x) = (3x+7)(4x^2+6x)$ **[2 marks]**

$f'(x) = (4x^2+6x).3 + (3x+7)(8x+6)$ ✓ applies and substitutes correctly into P.R.

$= 12x^2+18x+24x^2+18x+56x+42$

$= 36x^2+92x+42$

$= 2(18x^2+46x+21)$ } accept either ✓ correct and simplified

c) $y = \sqrt[3]{x^2-x-1}$ **[3 marks]**

$u = x^2-x-1$ and $\frac{du}{dx} = 2x-1$

$y = \sqrt[3]{u}$ and $\frac{dy}{du} = \frac{1}{3}u^{-\frac{2}{3}}$ ✓ defines u

so $\frac{dy}{dx} = \frac{1}{3u^{\frac{2}{3}}} . (2x-1)$ ✓ subs into C.R.

$= \frac{2x-1}{3u^{\frac{2}{3}}}$

$= \frac{2x-1}{3(x^2-x-1)^{\frac{2}{3}}}$ ✓ correct

or by sight $y = \sqrt[3]{x^2-x-1} = (x^2-x-1)^{\frac{1}{3}}$

so $\frac{dy}{dx} = \frac{1}{3}(x^2-x-1)^{-\frac{2}{3}} . (2x-1)$

$= \left(\frac{2x}{3}-\frac{1}{3}\right)(x^2-x-1)^{-\frac{2}{3}}$

$= \frac{2x-1}{3(x^2-x-1)^{\frac{2}{3}}}$ ✓ rearranges

✓ subs into C.R.

✓ correct

QUESTION 2 **[10 marks]**

Consider the function $f(x) = 2x^3 + 3x^2 - 12x - 2$

a) Find the coordinates and nature of all stationary point(s), and point(s) of inflection **[5 marks]**

$f'(x) = 6x^2 + 6x - 12$

let $0 = 6(x^2 + x - 2)$

$= 6(x+2)(x-1)$

so $x = -2$ or 1 ✓ both

at $x = -2$, $y = 18$

$x = 1$, $y = -9$

so S.Ps at $(-2, 18)$ and $(1, -9)$

✓ x and y coordinates

As $f''(x) = 12x + 6$

when $x = -2 \Rightarrow f''(x) < 0$, so maxima at $(-2, 18)$

$x = 1 \Rightarrow f''(x) > 0$, so minima at $(1, -9)$

✓ with reasons

To find POI

$f''(x) = 12x + 6$

let $0 = 12x + 6$

$-6 = 12x$

$-\frac{1}{2} = x$

at $x = -\frac{1}{2}$, $y = \frac{9}{2}$

so POI at $(-\frac{1}{2}, \frac{9}{2})$ ✓ both

check for concavity by sign test or use of $f'''(x)$ ✓

b) Describe the behaviour of $f(x)$ as $x \to \pm\infty$ **[1 mark]**

$x \to \infty$, so $y \to \infty$

and $x \to -\infty$, so $y \to \infty$

✓ must get both correct

c) i) Determine $f(-3)$ **[1 mark]**

$f(-3) = 2(-3)^3 + 3(-3)^2 - 12(-3) - 2$

$= 7$ ✓

only answer will be ok.

ii) Determine $f(3)$ **[1 mark]**

$f(3) = 2(3)^3 + 3(3)^2 - 12(3) - 2$

$= 43$ ✓

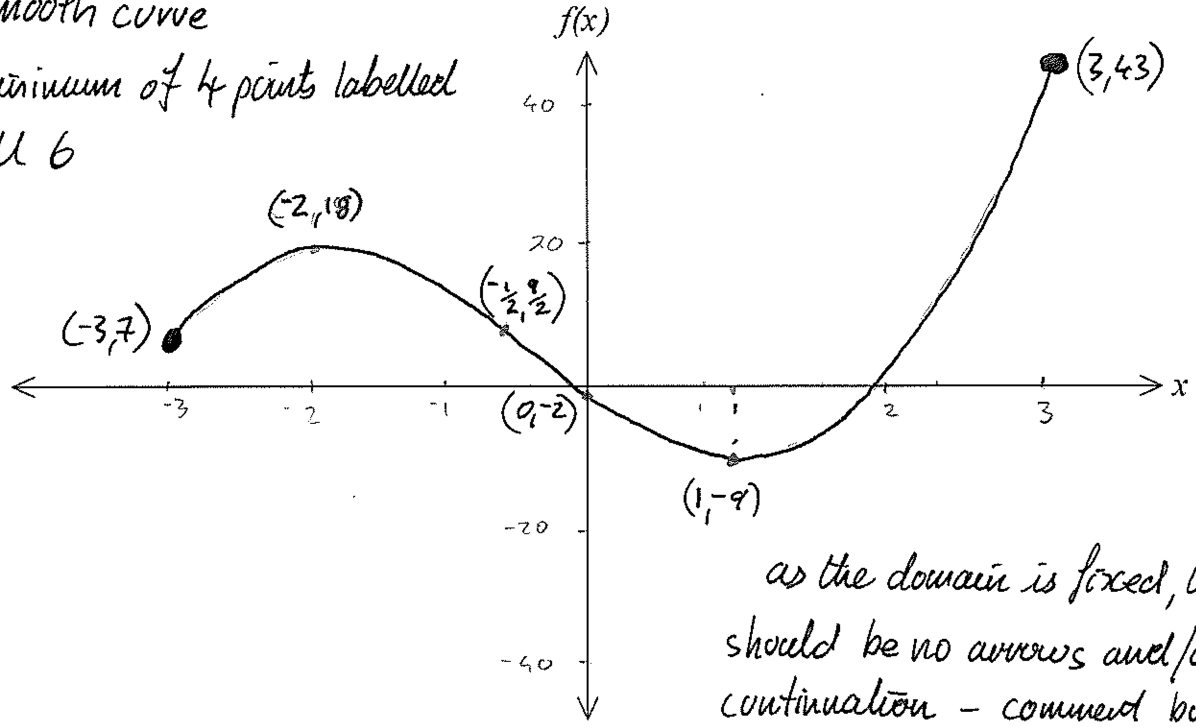
d) Applying your answers from parts a), b), and c), sketch $f(x)$ on the closed interval [-3,3] on the axes below labelling all relevant points.

[2 marks]

✓ smooth curve

✓ minimum of 4 points labelled

✓ all 6



as the domain is fixed, there should be no arrows and/or continuation – comment but don't penalise

QUESTION 3 **[3 marks]**

The two variables, p and q are related by the equation $p = \frac{2q-6}{q}$

a) Find an expression for $\frac{dp}{dq}$ **[1 mark]**

$p = \frac{2q-6}{q} = \frac{2q}{q} - \frac{6}{q} = 2 - 6q^{-1}$

$\frac{dp}{dq} = 6q^{-2} = \frac{6}{q^2}$

✓ correct expression accept other methods.

b) Hence, find an expression for the approximate increase in p, as q increases from 4 to 4+h, where h is small.

[2 marks]

$\frac{\delta p}{\delta q} \simeq \frac{dp}{dq}$

$= \frac{6}{q^2}$

$\delta p = \frac{6}{q^2} \cdot \delta q$

✓ subs into incremental formula.

let $q = 4$ $\quad = \frac{6}{4^2} \cdot h = \frac{6h}{16} = \frac{3h}{8}$

✓ correct value simplified.

MATHEMATICS METHODS : UNITS 3 & 4, 2021

Test 1 – (10%)

3.1.1 to 3.1.16

Time Allowed 30 minutes	**First Name**	**Surname**	**Marks** 27 marks

Circle your Teacher's Name: Mrs Alvaro, Mrs Bestall, Ms Chua, Mr Gibbon, Mrs Greenaway, Mr Luzuk, Mrs Murray, Ms Robinson, Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART B – CALCULATOR ASSUMED

QUESTION 4 **[3 marks]**

Given that the value v (\$) of a particular mineral is tied to its mass M (g) and is satisfied by the equation $v = 510M^{\frac{3}{4}}$, use the incremental formula to find the approximate value of an 8.01gram sample.

let $\delta m = 0.01$

then $\frac{dv}{dm} \cdot \delta m$

if $\frac{dv}{dm} = \frac{3}{4} \times 510 m^{-\frac{1}{4}} = \frac{382.5}{m^{\frac{1}{4}}}$ ✓ correct derivative

so small change in value is given by

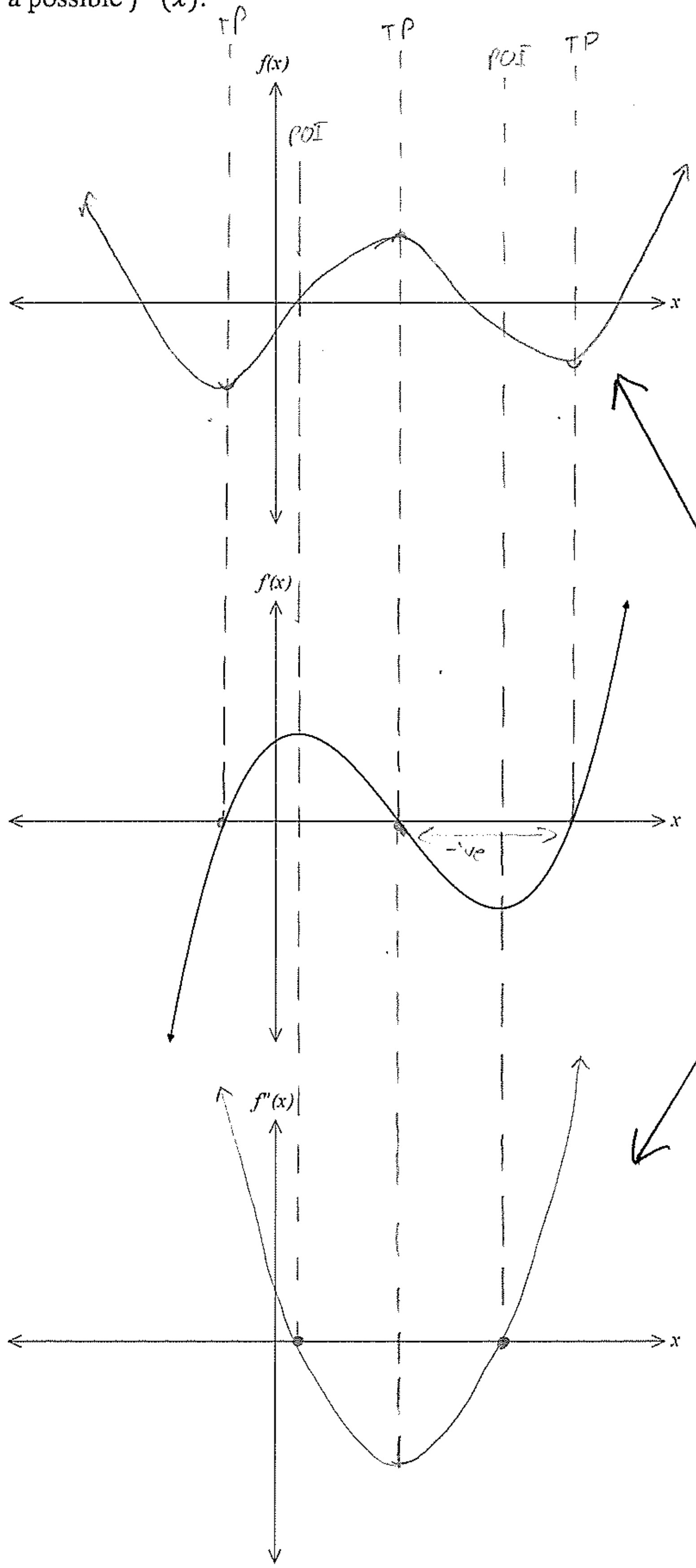
$\frac{382.5}{8^{\frac{1}{4}}} \times 0.01 \simeq 2.27\ldots$
$\simeq 2.27 \text{ (2d.p.)}$ ✓ subs into incremental formula and correct result.

so $510(8)^{\frac{3}{4}} + 2.27 = \$ 2428.25$

hence, the value of 8.01g will be \$2428.25. ✓

QUESTION 5 **[5 marks]**

The middle graph below represents the gradient function of $f(x)$, sketch on the top axes a possible $f(x)$, and on the bottom axes a possible $f''(x)$.



3×TP ✓

2×POI ✓

✓ 2× smooth curves and all +'ve/-'ve gradients correct.

2× x-int ✓

1× T.P ✓

QUESTION 6 **[6 marks]**

A body moves in a straight line so that its displacement, *s(t)* metres, from a point of origin after *t* seconds is given by $s(t) = t^3 - 9t^2 + 24t$, for $0 \leq t \leq 5$.

a) When is the body stationary? **[2 marks]**

$s'(t) = 3t^2 - 18t + 24$ ✓ differentiates correctly

when $0 = 3(t^2 - 6t + 8)$

$= 3(t-4)(t-2)$

hence, stationary at $t=2$ and $t=4$. ✓ both values of t

b) When is the body moving fastest? **[2 marks]**

$s'(5) = 75 - 90 + 24$

$= 9$ ✓

$s'(0) = 24$.

hence, moving fastest at $t=0$ ✓

c) Calculate the distance travelled by the body in the first 4 seconds. **[2 marks]**

$s(2) = 8 - 36 + 48$

$= 20$

✓ both correct

$s(4) = 64 - 122 + 96$

$= 16$

total distance $20 + 4 = 24m$ ✓

QUESTION 7 **[5 marks]**

Prove that the derivative of $y = \left(\frac{x^2-2}{x^2+1}\right)^4$ is given by $\frac{dy}{dx} = \frac{24x(x^2-2)^3}{(x^2+1)^5}$

by the Chain Rule

$$\frac{dy}{dx} = 4\left(\frac{x^2-2}{x^2+1}\right)^3 \times \frac{d}{dx}\left(\frac{x^2-2}{x^2+1}\right)$$

✓ sub into C.R.

by the Quotient Rule.

$$\frac{dy}{dx} = 4\left(\frac{x^2-2}{x^2+1}\right)^3 \times \left(\frac{(x^2+1).2x-[(x^2-2).2x]}{(x^2+1)^2}\right)$$

✓ sub into Q.R.

simplifying gives

$$\frac{dy}{dx} = 4\left(\frac{x^2-2}{x^2+1}\right)^3 \times \frac{2x^3+2x-2x^3+4x}{(x^2+1)^2}$$

✓ correct expansion

$$= 4\left(\frac{x^2-2}{x^2+1}\right)^3 \times \frac{6x}{(x^2+1)^2}$$

✓ correct

$$= \frac{24x(x^2-2)^3}{(x^2+1)^5}$$

✓ simplified expression as required

If different method used, follow through and mark accordingly.

QUESTION 8 **[3 marks]**

A pedantic child insists that the radii of all their spherical birthday balloons must be increased by 1%. Find the approximate percentage increase in volume of one such balloon.

$V = \frac{4}{3}\pi r^3$ and $\frac{dV}{dr} = 4\pi r^2$

given $\frac{100\delta r}{r} = 1$, need $\frac{100\delta V}{V}$

as $\delta V \simeq \frac{dV}{dr} \cdot \delta r$ ✓ sub into formula

$\simeq 4\pi r^2 \cdot \delta r$

$\therefore \frac{100\delta V}{V} = \frac{100 \cdot 4\pi r^2 \cdot \delta r}{V}$

$= \frac{100 \cdot 4\pi r^2 \cdot \delta r}{\frac{4}{3}\pi r^3}$ ✓

$= 3 \cdot \frac{100\delta r}{r}$

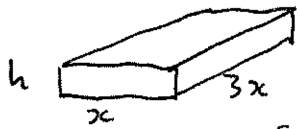
$= 3$

hence, the volume is increased by approximately 3%. ✓

QUESTION 9 **[5 marks]**

I want to construct a rectangular prism packing case from cardboard, with a lid, that will fully enclose an object whose length is three times its width x.

As the volume Vm^2 of the box is fixed, show that the area of cardboard required to make the case is a minimum when $x = \sqrt[3]{\frac{2V}{9}}$



$V = 3x^2 \cdot h$

so $\frac{V}{3x^2} = h$ ✓ isolates h.

$A = 2(3x^2 + 3xh + xh)$

$= 6x^2 + 8xh$

$= 6x^2 + 8x \cdot \frac{V}{3x^2}$

$= 6x^2 + \frac{8V}{3x}$ ✓ correct expression for A

$\frac{dA}{dx} = 12x - \frac{8}{3}Vx^{-2} = 12x - \frac{8V}{3x^2}$

solving for $0 = 12x - \frac{8V}{3x^2}$

$8V = 36x^3$

$\frac{8V}{36} = x^3$

$\frac{2V}{9} = x^3$ ✓

$\Rightarrow \sqrt[3]{\frac{2V}{9}} = x$ as required

✓ for $\frac{d^2y}{dx^2}$ test

To check minimum at $x = \sqrt[3]{\frac{2V}{9}}$

$\frac{d^2A}{dx^2} = 12 + \frac{16V}{3}x^{-3} = 12 + \frac{16V}{3x^3}$

let $x = \sqrt[3]{\frac{2V}{9}}$, so $12 + \frac{16V}{3\left(\sqrt[3]{\frac{2V}{9}}\right)^3} > 0$

as $\frac{d^2A}{dx^2} > 0$, so minimum ✓

! no actual need to calculate an answer as all positive values of x will be > 0